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Topological transitions at the resonance in gauge sector

Dmitri Grigoriev [1]

Dept. of Mathematical Physics, National University of Ireland,
Maynooth, Co. Kildare, Ireland

We discuss topological transitions during parametric resonance in the gauge sector of electroweak theory. It is shown that the resonance leads to separation of topological indices of the gauge and Higgs fields, resulting in topological transitions of non-sphaleron nature.

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[1] On leave of absence from Institute for Nuclear Research of Russian Academy of Sciences

1 Introduction

It has been suggested recently [2] that parametric resonance in the gauge sector of electroweak theory can substantially amplify baryoproduction due to CP-asymmetry present in the effective action and/or initial conditions. For many electroweak baryogenesis models, the baryoproduction is typically the product of CP-asymmetry times the mean shift of Chern-Simons number $N_{\rm CS}$ during out of equilibrium stage. In our scenario, $N_{\rm CS}$ becomes extremely high due to the resonance [1, 2, 3], getting parametrically close to unit charge per one sphaleron volume.

To get more specific estimate of baryoproduction [2], one needs to understand the dynamics of topology-changing processes in detail. In this talk we concentrate on the behaviour of topological indices during the parametric resonance in the gauge sector. We show that in our case the mechanism of topological transitions is considerably more complicated than in other scenarios of electroweak baryogenesis. Namely, while topological transitions changing both the Chern-Simons number of the gauge field and winding number of the Higgs field take place, during the resonance they never* follow standard sphaleron path found in [4, 5].

This point is illustrated at Fig. 1. The system states with minimal energies are located along the vacuum line $N_{\rm CS} = N_{\rm wind}$; in most existing electroweak baryogenesis scenarios CP-violating effects are driving both $N_{\rm CS}$ and $N_{\rm wind}$ along this line, with transitions between neighbouring topological vacua going along normal sphaleron path.

Unlike this common situation, the resonant amplification is a two-step process. Only the gauge field is directly involved into the resonance, so at first the system gets through the resonance into highly excited state with large topological index of gauge field (Chern-Simons number $N_{\rm CS}$) and small topological index of the Higgs field (winding number $N_{\rm wind}$, which is exactly 0 for ansatz used in numerical simulations of Ref. [2]). On $(N_{\rm CS}, N_{\rm wind})$ plane, Fig. 1, this state is quite a distance away from the line of vacuum states $N_{\rm CS} = N_{\rm wind}$. Of course, a lot of fermions is already created at this moment via the triangle anomaly due to large shift in $N_{\rm CS}$; however, none of them would survive the end of the resonance without topological transitions in Higgs sector which take place at the second stage of the process.

The second stage corresponds to the decay of the topological state created through the resonance. The most favourable path from the point with large $N_{\rm CS}$ and zero $N_{\rm wind}$ is obviously back along $N_{\rm wind} = 0$ line, but the gauge field dynamics in this direction is controlled by the resonance itself, and no transition at constant $N_{\rm wind}$ would create final state fermions anyway. However, any movement towards the vacuum line $N_{\rm CS} = N_{\rm wind}$ is energetically favourable; for relatively small $N_{\rm CS} - N_{\rm wind}$, winding number-changing transitions must go over energy barrier (e.g. through thermal or non-thermal activation), while at very large $N_{\rm CS}$ densities typical for gauge-sector resonance such transitions may

*To be precise, normal sphaleron transitions can occur, but they give zero contribution to the net baryoproduction.

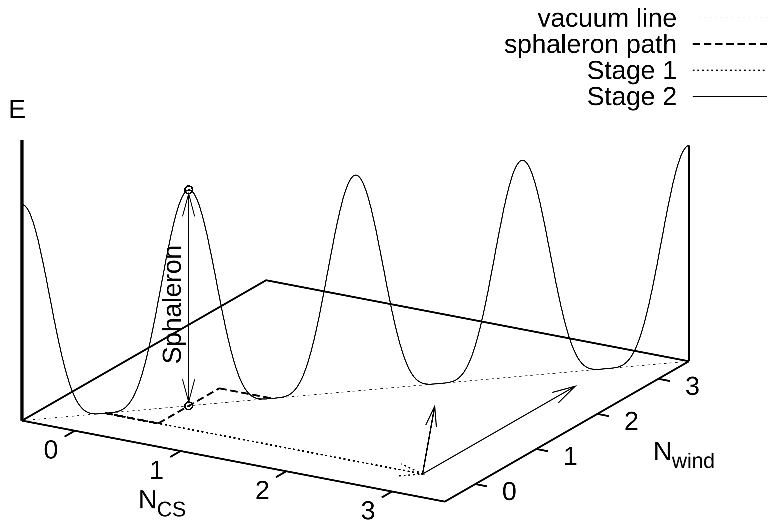

Figure 1: On $(N_{\rm CS}, N_{\rm wind})$ plane, the standard periodical vacuum structure exists along the vacuum line $N_{\rm CS} = N_{\rm wind}$; other directions give quite different energy profiles – see Ref. [2] and Figs. 2,3 below. During normal sphaleron transition, the system moves to the neighbouring vacuum via the sphaleron path, while in our case transition paths are more complicated (arrows).

become unsuppressed.

2 Stage I: the resonance in gauge sector

Homogeneous field ansatz used in [1, 2] makes things very simple here: gauge field $A_j = M_{\rm W}\phi\tau_j/2i$ is parametrised by single variable $\phi(t)$ with parabolic-like potential

$$E(\phi) = \frac{M_{\rm W}}{2g^2}\left(\phi^4 + \phi^2\right) \tag{1}$$

Anharmonic term in (1) limits the saturation amplitude of the resonance to be $\phi_{\rm max} \sim 1$, which corresponds to very high Chern-Simons number density of order of 0.1 per one sphaleron volume (provided the inflaton field has sufficient energy reserve). However, this density will turn into 0 and then change sign for every period of ϕ oscillations during the resonance. The end of the resonance would result in ϕ finally coming back to its zero vacuum value with zero Chern-Simons density, what makes all fermionic states created during the resonance to disappear.

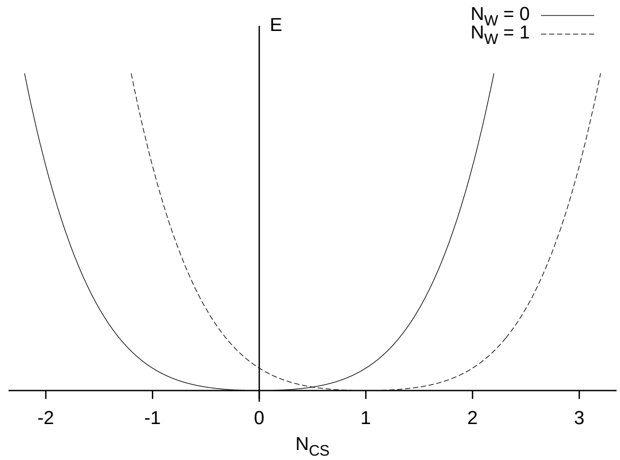

Figure 2: Energy profiles at fixed N_{wind} can be obtained [2] directly from Eq. (1). To get non-zero N_{CS} after the resonance, one has to change Higgs winding number (dashed line).

Therefore, creation of permanent fermionic states is equivalent to shifting the minimum point of potential $E(N_{\text{CS}})$ from 0, see Fig. 2. To do that, we need to change winding number of the Higgs field, once $N_{\text{CS}} = N_{\text{wind}}$ at the minimum. As we'll show below, topological transitions naturally occur in the Higgs sector at non-zero Chern-Simons number, although their dynamics is hard to investigate in detail.

3 Stage II: topological transitions changing the Higgs winding number

A general understanding of the dynamics of Higgs winding number-changing topological transitions can be obtained through the use of gauge invariance. Key argument here is that the total energy of the bosonic (gauge and Higgs) fields is invariant under large gauge transformations, which simultaneously change N_{CS} and N_{wind} by integer number N:

$$E(N_{\text{CS}}, N_{\text{wind}}) = E(N_{\text{CS}} + N, N_{\text{wind}} + N)$$

and, therefore,

$$E(N_{\text{CS}}, N_{\text{wind}}) = E(N_{\text{CS}} - N_{\text{wind}}, 0) \tag{2}$$

Note that the function $E(N_{\text{CS}}, 0)$ is known through Eq. (1) only for homogeneous gauge field, while the large gauge transformations generally break the homogeneous ansatz. This isn't an obstacle in (1+1) dimensions, where any field configuration can be transformed into gauge-equivalent one with homogeneous gauge field; however, one could expect that the subsequent analysis is also qualitatively applicable to (3+1) case at non-zero density of topological numbers, i.e. when the effects due to spatial localization of field configuration with unit topological charge are negligible.

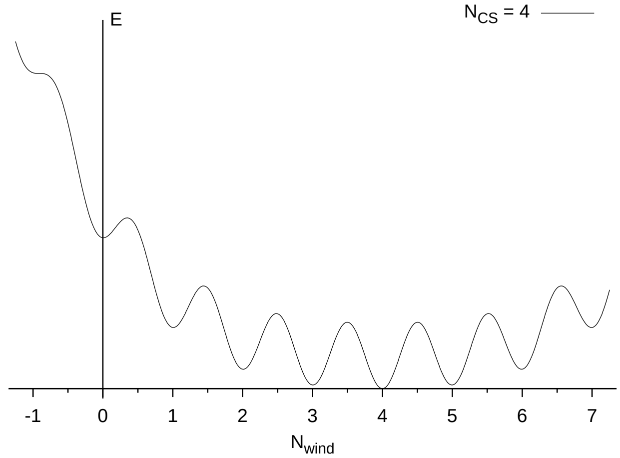

Figure 3: Energy profile at fixed $N_{\rm CS}$ is in principle identical to Fig. 2, see Eq. (2). However, integer values of $N_{\rm wind}$ are separated by sphaleron-like energy barriers; we expect them to disappear at large $|N_{\rm CS} - N_{\rm wind}|$.

Cross-section of $E(N_{\rm CS}, N_{\rm wind})$ in the direction orthogonal to Fig. 2 makes it obvious that non-zero Chern-Simons number indeed stimulates topological transitions in Higgs sector, once it is energetically favourable to reduce the difference between $N_{\rm CS}$ and $N_{\rm wind}$ (see Fig. 3). Of course, $N_{\rm wind}$ can take only integer values, so the continuous lines on Fig. 3 just demonstrate the presence of energy barriers that separate the neighbouring values of $N_{\rm wind}$. At $N_{\rm CS} = N_{\rm wind}$ the height of this barrier should be close to, but not identical to the sphaleron energy, once at Fig. 3 we are crossing the barrier at fixed integer $N_{\rm CS}$, while the sphaleron has half-integer $N_{\rm CS}$. Although the rigorous calculation of barrier heights and corresponding transition amplitudes at non-equal Chern-Simons and winding numbers is yet to be done even in (1+1) dimensions, it is natural to expect that the barrier in $N_{\rm wind}$ direction should vanish when the energy gain due to transition equals the sphaleron energy.

This means that topological transitions in Higgs sector should become very intensive at sufficiently large $N_{\rm CS}$; one can show [2] that it happens at $\phi \gtrsim 1$, i.e. at saturation amplitude of parametric resonance.

4 Conclusions

The main outcome of semiqualitative arguments presented above is that the parametric resonance in gauge sector moves us away from well-known paths in topological space. At both stages of the process the system never moves along the normal sphaleron path. At the first stage, the resonance drives the Chern-Simons number along the path with fixed winding number. Then the increase in $N_{\rm CS}$ makes the transitions with $\Delta N_{\rm wind}$ of the same sign more and more favourable until the transition towards the vacuum line finally takes place. The same two-stage process will happen at the next half-period of $N_{\rm CS}$ oscillations, when both $N_{\rm CS}$ and $\Delta N_{\rm wind}$ will have negative values. Even for one half-period, the actual trajectory of the system in topological space cannot be determined from static energy profiles. Improving estimates given in [2] for baryoproduction at parametric resonance in gauge sector will thus inevitably require better understanding of topological transitions at non-equal Chern-Simons and winding numbers, and more detailed numerical studies of their kinetics as well. Work in this direction is now in progress.

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